S18

Code No: F-15005

FACULTY OF SCIENCE

B.A./B.Sc. (CBCS) I-Semester Examination, December 2023/January 2024

Subject: Mathematics

Paper – I : Differential and Integral Calculus

Time: 3 Hours **Max. Marks: 80**

PART – A

(8x4= 32 Marks)

Note: Answer any eight questions.

1. Evaluate $\lim\limits_{\substack{x\to\infty \\ y\to 2}} \frac{xy}{x^2+2y^2}$
2. If $u = e^{xy}$ then find $\frac{\partial^2 u}{\partial y \partial x}$
3. If $u = e^{ax} sinby$ then find $\frac{\partial^2 u}{\partial x \partial y}$ and $\frac{\partial^2 u}{\partial y \partial x}$
4. If $z = e^{xy}$ where $x = t\,cost, y = t\,sint$ then find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$
5. Find $\frac{dy}{dx}$ if $(x)^y = (y)^x$
6. Find the stationary points of the function $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$
7. Find the radius of curvature at each point $P(\varphi, s)$ of the curve $s = c\,log(sec\varphi)$
8. Find the radius of curvature at the origin for the curve $x^3 + 3x^2y - 4y^3 + y^2 - 6x = 0$
9. Find the envelope of the families of the curve $\frac{x\,cos\alpha}{a} + \frac{y\,sin\alpha}{b} = 1$; where α is a parameter.
10. Find the length of arc of the catenary $y = c\,cosh\left(\frac{x}{c}\right)$ measured from the vertex to the point $P(x,y)$.
11. Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the $x-$ axis.
12. Find the surface area of the sphere of radius a

PART – B

(4 x 12 = 48 Marks)

Note: Answer all questions.

13. (a)(i) If $u = x^y$ then show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$

(ii) If $z = \tan(y+ax) + (y-ax)^{3/2}$ then find the value of $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}$

(OR)

(b)(i) If $u = \log(tanx + tany + tanz)$ then find the value of $sin2x\frac{\partial u}{\partial x} + sin2y\frac{\partial u}{\partial y} + sin2z\frac{\partial u}{\partial z}$

(ii) Verify Euler's theorem for $z = sin^{-1}\left(\frac{x}{y}\right) + tan^{-1}\left(\frac{y}{x}\right)$

14. (a)(i) If $z = f(x,y)$; $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$ then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$

(ii) If $u = x^2+y^2 + z^2$, where $x = e^t$, $y = e^t sint$ and $z = e^t cost$ then find $\frac{du}{dt}$

(OR)

(b) (i) Discuss the maxima or minima of the function $f(x,y) = 3x^2 - y^2 + x^2$

(ii) Expand the function $f(x,y) = e^x \log(1+y)$ in Taylor's expansion about $(0,0)$ up to second degree term.

15. (a) (i) For the curve $y = \frac{ax}{a+x}$, if 'ρ' is the radius of curvature of at any point $P(x,y)$ then show that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2$

(ii) Show that there is no envelope for the family of circles with centres $(\alpha,0)$ and radii α^2 where α is a parameter.

(OR)

(b) (i) Find the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$

(ii) Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are parameters connected by $a + b = c$

16. (a) (i) Find the length of the complete arch of the cycloid $x = a(\theta - sin\theta), y = a(1 - cos\theta)$.

(ii) Find the area of the surface of revolution generated by revolving about the y – axis the arc of $x = y^3$ from $y = 0$ to $y = 2$

(OR)

(b) (i) Find the perimeter of the cardioid $r = a(1 + cos\theta)$.

(ii) Find the volume of a spherical cap of height h cut-off from a sphere of radius $'a'$

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26/06/23 A/N

Code No: E-10005/BL

FACULTY OF SCIENCE

B.A. / B.Sc. (CBCS) I Semester (Backlog) Examination, June / July 2023

Subject: Mathematics

Paper – I: Differential and Integral Calculus

Time: 3 Hours **Max. Marks: 80**

PART – A

Note: Answer any eight questions. **(8 x 4 = 32 Marks)**

1. If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$ show that $x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0.$
2. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$
3. Verify that if $z = xy\, f(y/x)$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z.$
4. Find $\frac{dz}{dt}$ when $z = xy^2 + x^2y,\ x = at^2, y = 2at.$
5. If $F(x,y,z) = 0$ find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$
6. State Taylor's theorem for function of two variables.
7. Define radius of curvature.
8. Find the envelope of the straight lines $x\cos\alpha + y\sin\alpha = l\sin\alpha\cos\alpha$, α is parameter.
9. Find $\frac{ds}{dt}$ for the curve $x = a(t - sint), y = a(1 - cost).$
10. Find the perimeter of the circle $x^2 + y^2 = a^2.$
11. Find the length of the arc of the curve $y = logsec\, x$ from $x = 0$ to $x = \pi/4.$
12. Find the volume of the hemisphere.

PART – B

Note: Answer all the questions. **(4 x 12 = 48 Marks)**

13. a) State and prove Euler's theorem on homogeneous functions.

(OR)

b) If $u = \tan^{-1}\left(\frac{x+Y}{\sqrt{x}+\sqrt{y}}\right)$ show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{4}\sin 2u.$

14. a) Expand $\sin xy$ in powers of $(x - 1)$ and $\left(y - \frac{\pi}{2}\right)$ upto second degree terms.

(OR)

b) Prove that $f_{xy}(0,0) \neq f_{yx}(0,0)$ for the function f given by

$$f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2}; (xy) \neq (0,0),\ f(0,0) = 0.$$

15. a) Show that the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}.$

(OR)

b) Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$ when $ab = c^2,$ c is constant.

16. a) Find the volume of the right circular cone of height h and base of radius $a.$

(OR)

b) Find the length of the curve $y = \log\frac{e^1-1}{e^x+1}$ from $x = 1$ to $x = 2.$

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Code No: E-10005

FACULTY OF SCIENCE

B.A. / B.Sc. (CBCS) I - Semester Examination, February / March 2023

Subject: Mathematics

Paper – I: Differential & Integral calculus

Time: 3 Hours **Max. Marks: 80**

PART – A

Note: Answer any eight questions. **(8 x 4 = 32 Marks)**

1. If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$; $xy \neq 0$ prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$.
2. If $u = 3(lx + my + nz)^2 - (x^2 + y^2 + z^2)$ and $l^2 + m^2 + n^2 = 1$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.
3. If $Z = f(x + ay) + Q(x - ay)$ Prove that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.
4. If Z is a function of x and y and if $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ Prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.
5. Verify Euler's theorem for $z = ax^2 + 2hxy + by^2$.
6. If $x = u + e^{-v}\sin u$, $y = v + e^{-v}\cos u$ Prove that $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$.
7. Find $\frac{ds}{dx}$ for the curve $y = \cosh\left(\frac{x}{c}\right)$.
8. Define radius of curvature.
9. Obtain the envelope of the family of straight lines. $y = mx + \frac{a}{m}$.
10. Find the length of the circumference of the circle $x^2 + y^2 = 16$.
11. Find the whole length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.
12. Find the volume of the sphere of radius a.

PART – B

Note: Answer all the questions. **(4 x 12 = 48 Marks)**

13. (a) If $H = f(y - z,\ z - x,\ x - y)$, Prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$.

(OR)

(b) If $u = \cot^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{4}\sin 2u = 0$.

14. (a) Determine maximum or minimum values of u if $u = x^3 + y^3 - 3axy$.

(OR)

(b) Expand $f(x, y) = x^2 + xy - y^2$ by Taylor's theorem in powers of $(x - y)$ and $(y + 2)$.

15. (a) Obtain the evolute of the parabola $y^2 = 4ax$.

(OR)

(b) Find the envelope of the lines $\frac{x}{a} + \frac{y}{b} = 1$, when the parameters a and b are connected by the relation $a + b = c$.

16. (a) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $y = 3x$.

(OR)

(b) Find the volume of the right circular cone of height h and base of radius a.

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Code No. 6005

FACULTY OF SCIENCE

B.Sc. / BA (CBCS) I – Semester Examination, March 2022

Subject: Mathematics

Paper-I: Differential & Integral Calculus

Time: 3 Hours **Max. Marks: 80**

PART – A

Note: Answer any eight questions. **(8 x 4 = 32 Marks)**

1. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ does not exist.

2. If $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & if\ (x,y)\neq(0,0) \\ 0 & if\ (x,y)=(0,0) \end{cases}$

   then show that f is discontinuous at $O(0,0)$

3. If $f(x,y) = (x^2+y^2)e^{x-y}$ then evaluate $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y\,\partial x}$.

4. If $f(x,y) = \text{Cos}^{-1}\left(\frac{y}{x}\right)$ then find the total differential of f.

5. If $z = z(x,y) and\ x = e^{2u} + e^{-2v},\ y = e^{2u} - e^{-2v}$, then evaluate $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}$.

6. If $x^y = y^x$ then find $\frac{dy}{dx}$.

7. Find the radius of curvature of the curve $y = x^4 - 4x^3 - 18x^2$ at $O(0,0)$.

8. Find the radius of curvature of the curve $x = y^2 + 4y + 3$ at $P(8,1)$.

9. Find the envelope of the curve $y = mx + 2m^3$ where m is the parameter.

10. Find the length of the curve whose parametric equation is

    $x = e^{-t}\, cost, y = e^{-t}\, sint,\ \ 0 \le t \le \frac{\pi}{4}$.

11. Find the length of the arc of the curve $y = \log\tanh\left(\frac{x}{2}\right)$ from $x = 1\ to\ x = 2$.

12. Find the volume of the paraboloid generated by the revolution of the parabola $y^2 = 12x$ about the $x - axis$ from $x = 0\ \ to\ x = 5$.

PART – B

Note: Answer any four questions. **(4 x 12 = 48 Marks)**

13. (i) If $u(x,y,z) = \log(tanx + tany + tanz)$ then evaluate

    $$(sin2x)\frac{\partial u}{\partial x} + (sin2y)\frac{\partial u}{\partial y} + (sin2z)\frac{\partial u}{\partial z}.$$

    (ii) If $z(x,y) = (x+3y)^{\frac{3}{2}} + (x-4y)^{\frac{7}{2}}$ then evaluate $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 12\frac{\partial^2 z}{\partial y^2}$.

14. (i) If $z(x,y) = \text{Sec}^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2\, cotz.$

    (ii) If $u(x,y) = Tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then evaluate $x\,\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y}$.

15. If $f(x,y) = \begin{cases} \frac{xy(15x^2-7y^2)}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$ then show that $f_{xy}(0,0) \neq f_{xy}(0,0)$.

16. Find the extreme value of $f(x,y,z) = xyz$ when $x + y + z = 12$.

17. Find the circle of curvature of the curve $y^2 = 4ax$ at $P(am^2, 2am)$.

18. Find the evolute of the curve $x^2 = 4ay$.

19. Find the length of the loop of the curve $9y^2 = (x-2)(x-5)^2$.

20. Find the area of the surface of the solid generated by the revolution of an arc of curve $y = c\cosh\left(\frac{x}{c}\right)$ about $x - axis$.

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Maths

Code No. 19007

FACULTY OF SCIENCE

B.Sc. / B.A. I Semester (CBCS) Examination, August 2021

Subject: Mathematics - I

Paper – Differential and Integral Calculus

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 4 = 20 Marks)**

1 Evaluate $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$.

2 If $w = \log_e(x^2+y^2)$ find $\frac{\partial w}{\partial x}, \frac{\partial x}{\partial y}$.

3 State Euler' theorem and verify it for $(ax+by)^{\frac{3}{2}}$.

4 Find $\frac{du}{dt}$ when $u = \sin\left(\frac{x}{y}\right)$, $x = e^t$ and $y = t^2$.

5 Find the derivative of $f(x,y) = x^3 + y^3 - 3ax^2 = 0$.

6 Expand $f(x,y) = x^2 + xy + y^2$ in powers of $(x-1)$ and $(y-2)$ using Taylor's theorem.

7 Find the radius of curvature for the equation $x^3 + xy^2 - 6y^2 = 0$ at $(3,3)$.

8 Find the centre of curvature for the equation $xy(x+y) = 2$ at $(1,1)$.

9 Find the envelope of the family of curves $y = mx + \frac{a}{m}$ with m as parameter.

10 Find the length of the curve $y = \log_e(e^x+1) - \log_e(e^x-1)$ from $x = 1$ to $x = 2$.

11 Find the volume of the solid of revolution generated by revolving the plane area bounded by the curve $y = x^2$, $x = 3$ and X-axis.

12 Find the surface area of a sphere of radius 'r'.

PART – B

Note: Answer any three questions. **(3 x 20 = 60 Marks)**

13 Discuss the continuity of the function $f(x,y) = \begin{cases} \frac{x^2}{\sqrt{x^2+y^2}} & if \quad (x,y) \neq (0,0) \\ 3 & if \quad (x,y) = (0,0) \end{cases}$ at origin.

14 If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, $x^2+y^2+z^2 \neq 0$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

15 If $V = r^m$ where $r^2 = x^2+y^2+z^2$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = m(m+1)r^{m-2}$.

16 Find the minimum value of $x^2 + y^2 + z^2$ given that $ax + by + cz = p$.

17 Find the evolute of the curve $x = a\left(\cos t + \log \tan \frac{t}{2}\right), y = a \sin t$.

18 Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are connected by the relation $ab = c^2$, c being a constant.

19 Find the length of the Astroid $x^{2/3} + y^{2/3} = a^{2/3}$ where a is a constant.

20 Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about major axis.

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Code No. 19007/N/BL

FACULTY OF SCIENCE

B.A. / B.Sc. I Semester (CBCS) Examination, November/December 2021

Subject: Mathematics
Paper – I : Differential and Integral Calculus

Time: 2 Hours | Max. Marks: 80

PART – A

Note: Answer any five questions. (5 x 4 = 20 Marks)

1 Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2}$.

2 If $w = Tan^{-1}\left(\frac{y}{x}\right)$ find $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$.

3 Define homogeneous function and give two examples.

4 Find the total derivative of $u(x,y,z) = e^{xyz}$.

5 Find the derivative of $f(x,y)=(\cos x)^y-(\sin y)^x=0$

6 Expand $f(x,y)=e^y \log_e(1+x)$ in powers of x and y at (0,0) using Taylor's series.

7 Find the radius of curvature for the equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$.

8 Find the centre of curvature for the equation $x^3+y^3=2$ at (1,1).

9 Find the envelope of the family of curves $y=mx+\sqrt{1+m^2}$ with m as parameter.

10 Find the length of the curve $y=x\sqrt{x}$ from $x=0$ to $x=\frac{4}{3}$.

11 Find the volume of the solid of revolution generated by revolving the plane area bounded by the curve $y=x^3$, $y=0$, $x=2$ about x-axis.

12 Find the surface area of a sphere of radius 'a'.

PART – B

Note: Answer any three questions. (3 x 20 = 60 Marks)

13 Show that $f(x,y)=\begin{cases} \frac{x^3-y^3}{x^2+y^2} & when \ (x,y)\neq(0,0) \\ 0 & when \ (x,y)=(0,0) \end{cases}$ is continuous at origin.

14 If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=Tan^3 u$.

15 If $x=u+v$, $y=uv$ and $z=f(x,y)$ then prove that $u\frac{\partial z}{\partial u}+v\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}+2y\frac{\partial z}{\partial y}$.

16 Find the maximum value of $x^2+y^2+z^2$ given that $xyz=a^2$.